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A three-level non-deterministic modeling methodology for the NVH behavior of rubber connections

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ABSTRACT

Complex built-up structures such as vehicles have a variety of joint types, such as spot-welds, bolted joints, rubber joints, etc. Rubber joints highly contribute to the nonlinear level of the structure and are a major source of uncertainties and variability. In the general framework of developing engineering tools for virtual prototyping and product refinement, the modeling of the NVH behavior of rubber joints involve the computational burden of including a detailed nonlinear model of the joint and the uncertainties and variability typical of that joint in a full-scale system model. However, in an engineering design phase the knowledge on the joint rubber material properties is typically poor, and the working conditions a rubber joint will experience are generally not known in detail. This lack of knowledge often do not justify the computational burden and the modeling effort of including detailed nonlinear models of the joint in a full-scale system model.

Driven by these issues a non-deterministic numerical methodology based on a three-level modeling approach is being developed. The methodology aims at evaluating directly in the frequency domain the sensitivity of the NVH behavior of a full-scale system model to the rubber joint material properties when nonlinear visco-elastic rubber material behavior is considered. Rather than including directly in the model a representation of the rubber nonlinear visco-elastic behavior, the methodology proposes to model the material nonlinear visco-elastic behavior by using a linear visco-elastic material model defined in an interval sense, from which the scatter on the full-scale system NVH response is evaluated. Furthermore the development of a multi-level solution scheme allows to reduce the computational burden introduced by the non-deterministic approach by allowing the definition of an equivalent linear interval parametric rubber joint model, ready to be assembled in a full-scale system model at a reasonable computational cost.

By using a commercial finite element code the developed methodology is illustrated through a numerical case-study: the low-frequency dynamic analysis of automotive door weather-strip seals.

1. Introduction

In complex built-up structures such as vehicles, rubber joints are widely employed both for structural (i.e. engine mounts) and non-structural parts (i.e. drive shaft boots). Mounts, bushings, gaskets, boots, adhesives, seals, etc. are made in

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general of different rubber materials and are often characterized by complex geometries. Rubber joints are a major source of strong nonlinearities: geometrical, contact and material nonlinearities. Geometrical nonlinearities are due to the joint complex geometry and the large deformations they are subjected to in operational conditions. Contact nonlinearities are due to friction and due to the changing of contact conditions related to large deformations. Material nonlinearities are related to the nonlinear stress–strain material law typical for rubber materials, to the nonlinear visco-elastic material behavior, to the temperature dependency of material behavior, to material aging, tearing, etc.

In the general framework of developing engineering tools for virtual prototyping and product refinement, the finite element (FE) method is a well established approach for the solution of complex problems that are not readily tractable by classical analytical methods. Commercial FE codes allow the analysis of complex structures involving nonlinearities due to large geometrical deformations as well as contact problems. However, the treatment of problems involving nonlinearities due to material behavior is still an open issue. While the constitutive models for the three-dimensional behavior of rubber nonlinear elasticity as well as the rubber linear visco-elasticity are implemented in commercial FE codes, the rubber nonlinear visco-elastic constitutive models as well as their implementation in FE codes is still an open research field.

The interest in this study is on a particular nonlinear dynamic rubber material behavior known as Payne's effect [1,2], which is typical of filled rubbers (e.g. rubbers filled with carbon black particles). The Payne effect is observed under dynamic loading conditions, and it is manifested as a dependency of the material visco-elastic storage modulus on the amplitude of the applied dynamic strain: the storage modulus decreases asymptotically with increasing amplitude, reaching a lower bound at sufficiently large dynamic strain amplitudes.

The Payne effect can be attributed to the irreversible slip process between the rubber filler particles and their plastic deformations (see, e.g. the work of Miehe et al. [3] and Kaliske et al. [4]). Several modeling approaches have been proposed mostly considering this phenomenon as describable through chains of Coulomb friction elements (see, e.g. the work of Gregory [5], Austrell et al. [6], Berg [7], and Sjöberg et al. [8,9]).

These approaches present three types of problems: the calibration of the parameters describing the chains of friction elements are not easily determined from tests; the models are in the time domain, and for frequency domain applications, they involve additional computational burden; the models are mostly formulated for uniaxial cases and their implementation in FE codes does not seem immediate. A possible solution was proposed by Gil-Negrete et al. [10] by providing an adequate material data set in a linear visco-elastic constitutive model by enforcing an equivalent dynamic strain value.

In this study a different approach is proposed. It is observed that the strain amplitudes of the vibration a rubber joint experiences in real-life conditions are generally not known in detail. Not known a priori, which is a typical situation in a rubber joint early design phase, and not known a posteriori either as these strain amplitude levels generally depend on the modal behavior of the surfaces of the structure the rubber joint is attached to. This lack of knowledge often does not justify the computational burden and the modeling effort of including detailed nonlinear models of the joint in a full-scale system model.

When the aim is to evaluate the sensitivity of the full-scale system NVH response to a change in its rubber joint material properties this study proposes to include such an effect in a non-deterministic manner, by defining the parameters which identify a linear visco-elastic rubber material model in an interval sense. In this manner a linear visco-elastic approach can be adopted in each interval, and the sensitivity of the NVH response of the full-scale system model can be evaluated. The advantage of this approach is twofold: it allows to work directly in the frequency domain; and it allows to use commercial FE codes, where linear visco-elasticity is generally implemented.

As the inclusion of a detailed model of the joint into a full-scale system model is already computationally expensive, predicting the effect that an interval description of the material properties have on the full-scale system behavior becomes impractical, and alternative strategies are needed.

The methodology developed in this study is an interval non-deterministic solution procedure, which is based on a three-level modeling approach involving a material, a component and a system level. The idea behind the multi-level scheme is to reduce the computational burden introduced by the non-deterministic approach, by evaluating the effects of the propagation of the interval description of the material properties defined at material level through the component and the system level. In the component level the detailed model of the joint is substituted by an equivalent linear parametric model, and the intervals identified on the detailed model, are transformed into intervals on the parameters which define the equivalent model. Assembling the equivalent linear interval parametric model at system level results in a reduction of the computational cost of the overall solution procedure.

Starting from an interval description of the material properties at material level, different methods can be used to obtain bounds on the system response (see, e.g. the work of Moens et al. [11]). In this study, a multi-level design of experiment approach is chosen: the intervals identified at material level are sampled, and the samples propagated through the component and system levels to obtain the bounds on the system response. Although the bounds on the system response obtained in this manner are not exact, the general behavior of the system can be predicted and the correlation between the parameters investigated.

The developed methodology is a general non-deterministic numerical methodology that can be applied to any type of rubber joint. Besides the treatment of material nonlinearities, the methodology also allows to consider geometrical nonlinearities, contact problems and more in general uncertainties and variability typical of rubber joint applications.

The methodology is illustrated through a case-study which was previously investigated by the authors [12]: the low-frequency dynamic analysis of automotive door weather-strip seals. The case-study shows in particular the advantages of using the developed non-deterministic three-level methodology when predicting the behavior of systems which present rubber line joints.

The following section discusses in general terms the step-by-step solution algorithm of the developed non-deterministic numerical methodology. In Section 3 the case-study chosen to illustrate the developed methodology is discussed. The techniques used to model the rubber nonlinear quasi-static as well as the linear visco-elastic material behavior are discussed in detailed. In Section 4 the proposed non-deterministic approach to model the nonlinear visco-elastic rubber material behavior is explained. In Section 5 a direct solution scheme is defined where the non-deterministic approach is applied directly to the full-scale system model of the case-study with the weather-strip seal modeled in detail. In Section 6 the three-level methodology, developed to reduce the computational burden introduced by the proposed non-deterministic approach, is applied to the case-study. Each level of the developed methodology is discussed in detail, with emphasis given to the discussion on the reduction of the detailed weather-strip seal model to the equivalent linear interval parametric joint model. The section ends with a comparison in terms of accuracy and computational cost between the direct solution scheme and the three-level methodology. In the last section a summary of the results is shown and a discussion on the developed numerical methodology, its limitations and possible improvements is presented.

2. The non-deterministic three-level numerical modeling methodology

Every solution procedure requires the knowledge of a minimum of information concerning the material used, the geometry and boundary conditions of the component under analysis, and the uncertainties and variability typical of the application.

As the inclusion of a detailed model of the joint into a full-scale system model is already computationally expensive, predicting the effect that a non-deterministic description of a joint property has on the full-scale system behavior becomes impractical, and alternative strategies are needed.

The developed non-deterministic numerical methodology proposes a multi-level solution procedure which consists of three levels: a material, a component and a system level, and works directly in the frequency domain. The procedure starts from a non-deterministic description of the material properties at material level. The propagation of this non-determinism through the component and system level is studied to obtain the scatter in the desired system response. At component level the detailed model of the joint is reduced to an equivalent linear parametric model, and the scatter identified on the properties of interest of the detailed model is transformed into a scatter on the parameters which define the equivalent model, resulting in a reduction of the computational cost of the solution procedure.

A step-by-step description of the developed numerical methodology is shown in Fig. 1. We assume that a rubber joint of geometry G experiences in working conditions a range of temperatures ΔT, that works in a range of frequencies Δf, and that is subjected to a range of vibration amplitudes Δa. Depending on the range of vibration amplitudes Δa, the joint experiences a range of strain amplitudes $\Delta\varepsilon$, and is subjected to a variation in its geometrical properties ΔG_a. Considering the change of joint geometrical properties due to static pre-load ΔG_{pr}, the total change of joint geometrical properties will be $\Delta G = \Delta G_{pr} + \Delta G_a$. A step-by-step description of the developed numerical methodology is given below:

- *Material level*: In this level a material constitutive model is chosen and its parameters C_i identified. Assuming the material properties of interest are measured within the defined ranges of ΔT, Δf and $\Delta\varepsilon$, the corresponding non-determinism on the parameters defining the material constitutive model ΔC_i is identified.
- *Component level*: This level consists of two steps. In the first step a detailed model of the component is built-up, and a component property of interest is chosen, e.g. the stiffness of the component in a preferred direction K_Z. The non-determinism on the parameters defining the material constitutive model ΔC_i at material level is propagated to the component level, resulting in a scatter on the component properties of interest. Considering the non-determinism on the component geometry ΔG results in a wider range of variation for the component property of interest ΔK_Z.
  In the second step an equivalent linear parametric component model defined by k_i parameters is developed. The non-determinism on the component properties ΔK_Z is transformed into a scatter Δk_i on the equivalent model parameters.
- *System level*: In the last level the linear equivalent interval parametric component model is assembled in the full-scale system model, and the scatter on a system response of interest ΔR (e.g. a system frequency response function of reference) is evaluated from the non-determinism defined on the equivalent component parameters Δk_i.

Over the past years, interval methods have become increasingly popular for the representation of non-determinism in numerical analysis. Many researchers have focused on the problem of propagating interval uncertainties at the input side of a problem to the corresponding ranges of possible output behavior at the output side for many types of problems. Especially in the framework of finite element modeling, numerous techniques have been studied over the last decade, see e.g. the work of Moens et al. [13], Chen et al. [14], Rao et al. [15] and Muhanna et al. [16]. These techniques are also often encountered in studies involving uncertainty representation through fuzzy sets, see, e.g. the work of Rao et al. [17], Möller et al. [18], Hanss [19], Rama Rao et al. [20] and De Gersem et al. [21].

In this study an interval description of the non-determinism is considered. Starting from an interval description of the material properties at material level, different methods can be used to obtain bounds on the system response (see for example the work of Moens et al. [11]).

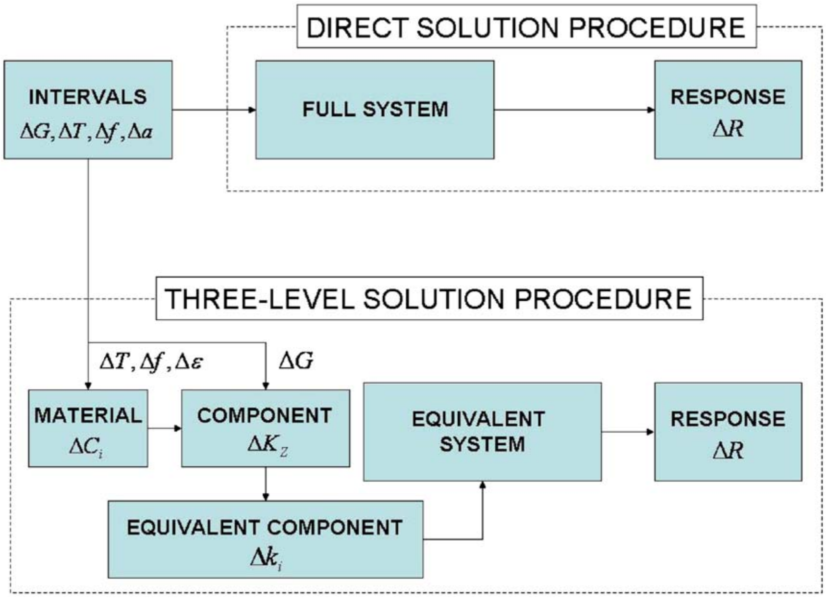

Fig. 1. Block diagram of the developed non-deterministic three-level methodology, comparison with a direct solution procedure where non-determinism is considered directly at full-scale system level.

For FE analysis a clear distinction can be made between two fundamental approaches: the global optimization approach and the interval arithmetic strategy approach. As the system behavior is frequency dependent, a global optimization approach would require the search for an optimal solution at each frequency line, i.e. an upper and lower bound on the system output, with the consequent increase of the computational burden. On the other hand, the use of interval arithmetics to propagate these intervals through the different levels could lead to an overestimation of the bounds on the system response, and more importantly to the risk of obtaining non-physical solutions. In addition some application-specific interval finite element solution schemes (see for example the works of Moens et al. [22] and JongSok et al. [23]) would encounter difficulties in dealing with systems involving nonlinearities, frequency dependent material behavior and localized damping.

In this study, a multi-level design of experiment approach is chosen. The intervals identified at material level are sampled, and the samples are propagated through the component and system levels to obtain the bounds on the system response. Although the bounds on the system response obtained in this manner are not exact, the general behavior of the system can be predicted (e.g. the creation of a system response surface of interest), and the correlation between the parameters investigated.

3. The low-frequency dynamic analysis of automotive door weather-strip seals

The developed three-level methodology is illustrated through a case-study, which represents a typical automotive NVH problem: the low-frequency dynamic analysis of automotive door weather-strip seals.

Automotive weather-strip seals are typically extrusion bulbs made of rubber material that are attached to either the car door or the car body in order to seal the passenger compartment. When closing the car door onto the car body, the door remains in contact with the body through the hinges at the front side, through the lock mechanism at the rear side and through the seal strip all around the car door perimeter. The seal strip induces some residual stiffness and visco-elastic contribution to the car door support condition and it plays a role in the way exterior noise and vibration is transmitted to the interior of the car.

In the static analysis of the mechanical characteristic of weather-strip seals, Wagner et al. [24] have shown the use of the nonlinear FE method for assessing performance factors that are considered in the automotive weather-strip seal design. In the low-frequency dynamic analysis of the mechanical characteristic of the weather-strip seals, Stenti et al. [12] defined the weather-strip seal structure-borne transmission characteristic as an additional performance factor to be considered in seal design for NVH applications.

Following the work of Stenti et al. [12], the developed non-deterministic multi-level numerical methodology is applied to the low-frequency dynamic analysis of the mechanical characteristic of a car door weather-strip seal.

3.1. Description of the case-study

The MSC.Marc nonlinear FE solver [25] is used to model the case-study; however, the choice of the solver does not limit the generality of the developed methodology.

The case-study considered is shown in Fig. 2, and consist of an aluminum plate of 136 mm length, 70 mm width and 2 mm thickness, which is clamped at one end and is connected along one side to the ground through a weather-strip seal. The weather-strip seal considered in this study is a typical section of a car door primary bulb weather-strip seal. The component has a total height of 30 mm, with the ring shape having an external diameter of 15 mm and a thickness varying between 2 and 3 mm. The weather-strip seal is in contact with the plate (the car door) through the upper ring and it is grounded through the fastener to a rigid surface (the car body). It is assumed that only the ring shape in the upper part, directly in contact with the plate, is contributing to the structure-borne transmission characteristic of the weather-strip seal. The fastener in the lower part is meshed only where contact with the upper ring is expected during compression. The fastener is made of two materials. A much stiffer material than the upper ring, *light* meshed area in Fig. 2, which is assumed an order of magnitude stiffer than the upper ring, and a small flap, *dark* meshed area in Fig. 2, which is made of the same material of the upper ring and is involved in the weather-strip seal compression during door closing. The plate is modeled via isoparametric 4-node thin shell elements with six degrees of freedom per node. The weather-strip seal is modeled via isoparametric 9-node brick elements, which show an extra pressure node taking into account the assumed incompressibility characteristic of the material [25]. The element size is chosen so that a satisfactory mesh refinement is obtained through the seal thickness and for the surface contact regions.

For the plate-seal and seal self-contact, a Coulomb friction model with a friction coefficient of 1.0 is used (see, e.g. [26,27]). However, the final deformed seal shape in the case-study is slightly influenced by the sliding of the contact surfaces.

A reference point frequency response function (FRF), evaluated at the bottom corner of the plate free edge, is used to investigate the sensitivity of the dynamic behavior of the clamped plate to a change in the rubber joint properties of interest: a change in the rubber joint pre-load ΔG; and a change in the rubber joint material properties due to a change in the amplitude Δa of the input vibration. The non-deterministic approach proposed in this study to model the rubber joint properties of interest is shown in Section 4.

The problem can be considered as a small amplitude vibration problem Δa (car door/body vibration), superimposed on a pre-loaded state ΔG (car door/body closure gap). It should be noted that in this study it is assumed that the amplitudes of vibration Δa generate only amplitudes of strain $\Delta\varepsilon$ and do not contribute to the change of the joint geometry (i.e. ΔG_a is negligible).

The case-study built as such is representative of a typical car door–body system problem.

3.2. Modeling the material nonlinear quasi-static behavior

The plate is made of aluminum, with Young's modulus $E = 72\,\text{GPa}$, Poisson coefficient $\nu = 0.33$ and density $\rho = 2800\,\text{kg}\,\text{m}^{-3}$. The material is assumed linear elastic, homogeneous and isotropic.

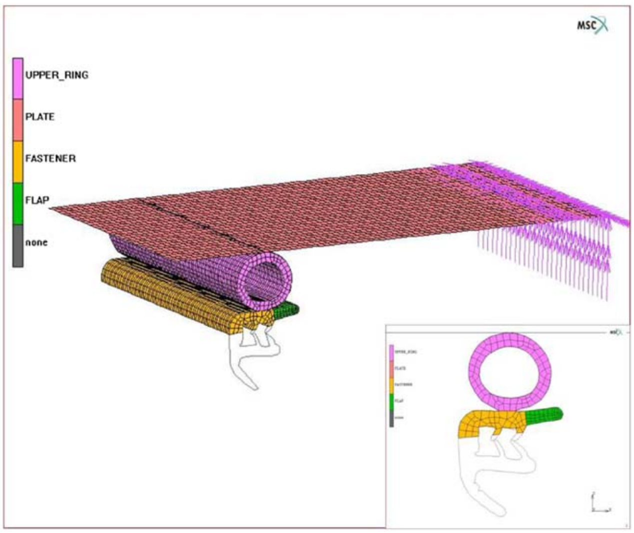

Fig. 2. The FE model of the case-study used to illustrate the developed three-level methodology.

The weather-strip seal is made of rubber. The measured rubber material density is $\rho = 340\,\mathrm{kg\,m^{-3}}$. The assumptions made in the work of Stenti et al. [12] in modeling the rubber material nonlinear quasi-static behavior as well as the rubber material linear visco-elastic behavior are considered valid also for this case-study, and are briefly reported here. It is assumed in all domains that the material is homogeneous, isotropic and incompressible. To model the material nonlinear quasi-static effects, an hyperelastic material model [28] is defined. Hyperelastic material models are characterized by their strain energy density function W. The hyperelastic material model chosen for the case-study is a Neo-Hookean material model, characterized by a strain energy density function W given by

$$W = C_{10}(I_1 - 3) \tag{1}$$

where $I_1 = \mathrm{tr}(B_{ij})$ is the first invariant of the left Cauchy–Green deformation tensor B_{ij}, $C_{10} = G^{\infty}/2$ is a material constant and G^{∞} is the equilibrium shear modulus evaluated from a static shear test.

3.3. Modeling the material linear visco-elastic behavior

The material linear dynamic behavior is modeled via the theory of small amplitude vibrations superposed on deformed visco-elastic solids, developed by Lianis [29], and based on the finite linear visco-elasticity theory of Coleman et al. [30]. Morman et al. [31] presented a simplified version of the theory, applicable to a certain class of incompressible materials which exhibit separability of time and large pre-strain effects, and discussed its implementation into the MSC.Marc nonlinear FE code. The theory results in the material constitutive tensor being represented as a complex fourth-order tensor L^{*}_{ijkl} defined in the undeformed configuration as

$$L^{*}_{ijkl} = D_{ijkl} + 2\mathrm{i}\omega\Phi^{*}_{ijkl} \tag{2}$$

In Eq. (2), D_{ijkl} is the static equilibrium nonlinear elastic material constitutive tensor defined as (the incompressible term is omitted for clarity)

$$D_{ijkl} = 4\frac{\partial^2 W}{\partial C^0_{ij}\partial C^0_{kl}} \tag{3}$$

where C^0_{ij} is the right Cauchy–Green deformation tensor expressed in the undeformed configuration. The term Φ^{*}_{ijkl} in Eq. (2) has a complicated expression which is not reported here (it can be found on the work of Morman et al. [31]). What is relevant to know for this study is that the term Φ^{*}_{ijkl} is a function of the normalized material complex shear modulus $g^{*}(\omega)$, defined as

$$g^{*}(\omega) = \frac{1}{2}\frac{G^{*}(\omega) - G^{\infty}}{\mathrm{j}\omega G^{\infty}} \tag{4}$$

In Eq. (4), $G^{*}(\omega)$ is the material complex shear modulus identified from measurements, which can be written as

$$G^{*}(\omega) = G_s(\omega) + \mathrm{j}G_l(\omega) \tag{5}$$

where G_s and G_l are the real and imaginary part of the complex shear modulus, and are known, respectively, as the storage and the loss shear modulus.

Since the quasi-static $G_s(\omega)$ modulus evaluated at very low frequencies can be identified with the equilibrium shear modulus G^{∞}, it is clear from the chosen material models that a single measurement of the material complex modulus $G^{*}(\omega)$ gives all the material parameters required by the numerical simulation [12].

It is important to emphasize that the linear visco-elastic rubber material model identified by Eq. (5) is valid for one amplitude of the excitation. For rubbers which are not filled (e.g. no addition of carbon black particles), the material complex modulus $G^{*}(\omega)$ is slightly dependent on the amplitude of the excitation [1,2]. For this type of rubbers the linear visco-elastic model in Eq. (5) is sufficient to describe the rubber material dynamic behavior, and a single measurement of the material complex modulus $G^{*}(\omega)$ at any amplitude of the excitation is sufficient to give all the material parameters required by the numerical simulation. For filled rubbers the visco-elastic behavior is nonlinear, and the material complex modulus $G^{*}(\omega)$ is dependent on the specific amplitude of the excitation used. When the amplitudes of the vibration a rubber joint experience in real-life working conditions are not known in detail, rather than representing the nonlinear visco-elastic material behavior directly (see for example [32,8]), it is proposed in this study to use a non-deterministic approach by defining intervals on the parameters identifying the model described by Eq. (5). The proposed non-deterministic approach is described in Section 4.

4. A non-deterministic approach to model the rubber material nonlinear visco-elastic behavior

In this section the non-deterministic approach proposed to model the rubber material nonlinear visco-elastic behavior is explained. As the pre-load state of a rubber joint in its real-life working conditions is also of typical concern for the rubber joint design engineers, the weather-strip seal closure gap distance is considered in this study as an additional rubber joint property of interest.

4.1. Modeling the Payne effect in an interval sense

The rubber material nonlinear visco-elastic behavior, also known as Payne's effect, is observed in filled rubbers under dynamic loading conditions, and it is manifested as a dependency of the rubber material visco-elastic storage modulus on the amplitude of the applied dynamic strain.

It is noted that the specific strain amplitude $\Delta\varepsilon$ of the vibration a rubber joint experiences in real-life conditions is generally not known a priori, as in a typical rubber joint initial design phase, and for this reason it can be considered by the design engineer as a modeling uncertainty. However even a posteriori it is dependent on the modal behavior of the surfaces of the structure the rubber joint is attached to, such that a complete reduction of this uncertainty to a deterministic value is not advisable and it is opinion of the authors that it should be always treated as a variability [11]. This lack of knowledge often do not justify the computational burden and the modeling effort of including a representation of the material nonlinear visco-elastic behavior in a full-scale system model.

When the aim is to evaluate the sensitivity of the full-scale system NVH response to a change in its rubber joint material properties this study proposes to include such an effect in a non-deterministic manner, by defining the parameters which identifies a linear visco-elastic rubber material model in an interval sense.

In this study it is assumed that the weather-strip seal rubber material experiences in real-life working conditions amplitudes of strain varying between [0.1–10] percent. Following the approach proposed by the authors in [12] a material dynamic shear test is performed in a frequency range between [0–400] Hz, and in a strain amplitude range of the excitation signal between [0.1–10] percent. The measured real part of the material complex shear modulus and the relative loss factor (defined as the imaginary over the real part of the material complex shear modulus) are given in Figs. 3 and 4, where it is observed a typical linear visco-elastic behavior at constant strain amplitude: the real part and the loss factor of the material complex shear modulus increase with frequency. In Figs. 5 and 6 it is shown at one specific frequency the variation of the real part and the loss factor of the material complex shear modulus for different dynamic amplitudes of the input vibration. It is observed a typical nonlinear visco-elastic behavior at constant excitation frequency: the real part of the material complex shear modulus decreases with increasing the strain amplitude of the excitation signal (in agreement with the Payne effect). It is noted from the measurements done on the specific rubber of interest that the imaginary part of the material complex shear modulus is less influenced by this effect, and thus as a consequence the loss factor increases with the strain amplitude of the excitation signal [12].

The measurements of the material complex shear modulus are used to identify the parameters which defines the material models chosen to describe the rubber material behavior in the quasi-static and in the dynamic domain. The material quasi-static behavior is described in Section 3 by the model defined by Eq. (1). This model is defined when the parameter $C_{10} = G^{\infty}/2$ is assigned. By inspecting Fig. 3 it is noted that all the measured curve tend to converge to a single value around 0 Hz. Therefore it is chosen $G^{\infty} = 0.08\,\mathrm{N\,mm^{-2}}$, which corresponds to the value of G_s in Eq. (5) measured at 1 Hz.

According to the specifications in the MSC.Marc manual, the theory of small amplitude vibrations superimposed on deformed visco-elastic solids is implemented via the user defined FORTRAN subroutine *UPHI*. For the visco-elastic model it is necessary to pass to the *UPHI* subroutine an analytical expression of $G^*(\omega)$. In this study a power functions of the form

$$G^*(\omega) = G_s(\omega) + \mathrm{j}G_l(\omega) = (\alpha + \beta\omega^{\gamma})_s + \mathrm{j}(\beta\omega^{\gamma})_l \tag{6}$$

is chosen to represent the linear visco-elastic behavior of the rubber material. However, it is important to recall that this model is valid only for one specific dynamic amplitude of the excitation signal.

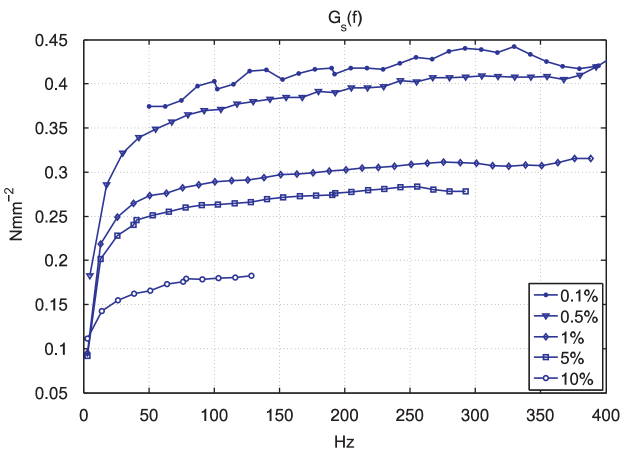

Fig. 3. Real part of the complex shear modulus measured for different values of the strain amplitude of the vibration.

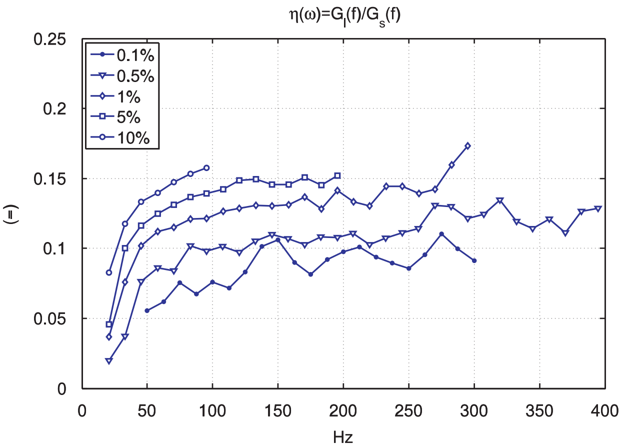

Fig. 4. Loss factor of the complex shear modulus measured for different values of the strain amplitude of the vibration.

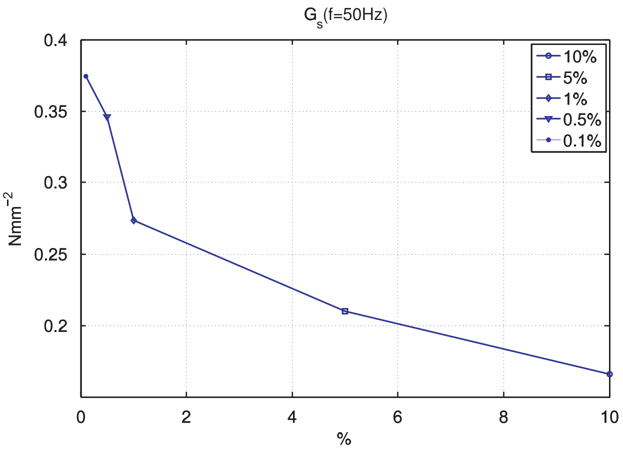

Fig. 5. Real part of the complex shear modulus measured at one specific frequency for different values of the strain amplitude of the vibration.

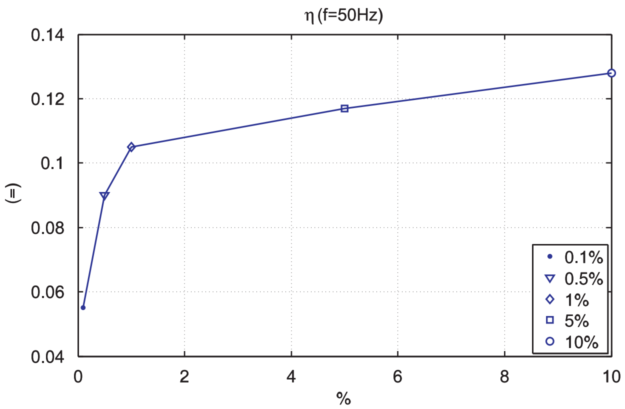

Fig. 6. Loss factor of the complex shear modulus measured at one specific frequency for different values of the strain amplitude of the vibration.

It is noted that the Payne effect results in a bounded behavior of the material storage modulus between an upper and a lower limit, which corresponds, respectively, to the smallest and the largest strain amplitudes below and above which the material storage modulus is not influenced by a change of strain amplitude [1]. Therefore, rather than modeling the Payne effect directly (see for example [32,8]), it is proposed in this study to include such an effect in a non-deterministic manner, by defining the parameters which identifies the previously defined linear visco-elastic rubber material model in an interval sense.

To model the dependency of the material storage and loss modulus from the strain amplitude, a translation of the material strain interval $\Delta\varepsilon$ to intervals on the real and imaginary part of Eq. (6) is performed.

It is noted that G^{∞} in Eq. (1) is represented by the value of α_s in Eq. (6). This parameter is not influenced by a change in the dynamic amplitude of the input vibration and thus it is considered independent from the Payne effect. The parameter α_s is thus set to $0.08\,\mathrm{N\,mm^{-2}}$.

To identify the parameters β_s, γ_s, β_l and γ_l of Eq. (6), a nonlinear least-square data-fitting problem is set up. Five different coefficient sets are identified, one for each dynamic amplitude of the input vibration considered. However, for the material storage modulus shown in Fig. 3, it is noted that the shape of the different curves is preserved as the amplitude of the excitation signal changes, and thus the parameter β_s—related to the storage part of the material complex shear modulus—is assumed as the only parameter which needs to be defined in an interval sense. For the material loss modulus, it is noted from the measurements done on the specific rubber of interest that the imaginary part of the material complex shear modulus is less influenced by the Payne effect, and thus that for the specific case the variation of the loss factor shown in Fig. 4 is due principally to the variation of the material storage modulus. Therefore, for the specific case it is assumed that the imaginary part of Eq. (6) is independent from the Payne effect. It is important to note that this assumption is not limiting the generality of the developed methodology, as it simply influences the number of parameters in Eq. (6) that needs to be described in a non-deterministic manner. With the assumptions made, the identified material parameters are as shown in Table 1, where α_s is fixed, γ_s, β_l, and γ_l are the average values of the previously identified five parameter sets, and β_s is the only parameter defined in an interval sense. The minimum and the maximum values identified for the parameter β_s define the interval on the material model parameter considered in this study.

The material modeling uncertainty $\Delta\varepsilon$ is thus translated into an interval on the material parameter $\Delta C_i = \Delta\beta_s$, and the nonlinear dynamic visco-elastic material behavior is modeled with a linear dynamic visco-elastic material model defined in an interval sense. In this manner a linear visco-elastic approach can be adopted in each interval and the sensitivity of the NVH response of the full-scale system model to a change in its rubber joint material properties can be evaluated. The advantage of this approach is twofold: it allows to work directly in the frequency domain; and it allows to use commercial FE codes where linear visco-elasticity is generally implemented.

4.2. Modeling the weather-strip seal closure gap distance

The second joint property of interest in this study is the weather-strip seal pre-load, which is determined once the car door–body closure gap distance is identified. When the car door is locked on the car body, the closure gap distance between the door and the body is determined. However in a first step system design phase, generally this information is not known a priori. In addition the closure gap distance is not constant along the door perimeter, and when considering building tolerances, it is required that the weather-strip seal model is analyzed for closure gap distances differing up to 3 mm from the nominal design closure gap distance [24]. The interval describing a variation on the weather-strip seal pre-load is considered to be describing a geometrical modeling uncertainty ΔG_{pr} which represent a tolerance on the closure gap distance between the car door and the car body. In this study the weather-strip seal closure gap level (CL) is defined as

$$\mathrm{CL} = \frac{(L_i - L_f)}{L_i} * 100 \tag{7}$$

where L_i and L_f are, respectively, the initial and final internal diameter of the component. The geometrical modeling uncertainty is modeled as an interval on the compression level: $\Delta G_{pr} = \Delta\mathrm{CL}$. In this study an interval on the compression level of [1–100] percent is considered.

4.3. Interpretation of the generated intervals

In this study a multi-level design of experiment approach is chosen. Eleven equally spaced samples are considered on each of the defined intervals, for a total of 121 samples for the simulation. The material variability $\Delta\beta_s$ and the geometrical modeling uncertainty ΔCL are identified as in Fig. 7.

In Fig. 8 the weather-strip seal compression load deflection curve in the direction normal (CLD-Z) and tangential (CLD-X) to the contact between the plate and the weather-strip seal is shown. It is apparent from the figure the nonlinear quasi-static behavior of the rubber joint. The domain defined in Fig. 7 can be interpreted by looking at Figs. 5 and 8. Each sample in the domain is representing a rubber joint whose quasi-static stiffness is identified by the tangent to the CLD curve at the correspondent CL (tangential stiffness), and whose dynamic stiffness is dependent on frequency and on the dynamic amplitude of the input vibration in agreement with the corresponding parameter β_s. The samples along the x-axis in Fig. 7

Table 1
The parameters of the power function fitted on the measured material complex shear modulus, where α and β have, respectively, the dimensions $\mathrm{N\,mm^{-2}}$, $\mathrm{N\,mm^{-2}(Hz)^{-\gamma}}$ and γ is a-dimensional.

α_s	$\Delta\beta_s$	γ_s	β_l	γ_l
0.08	[0.0192–0.1882]	0.1591	0.0151	0.293

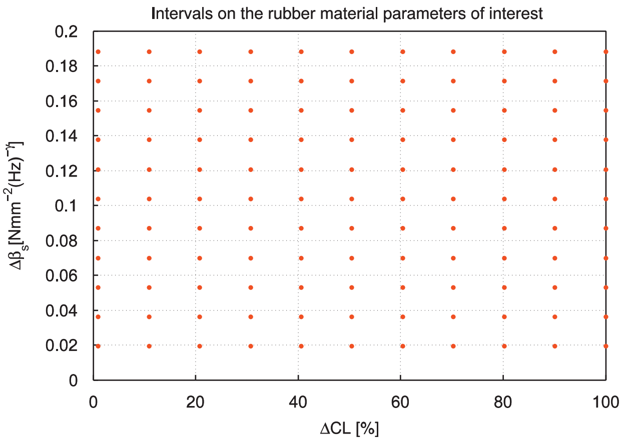

Fig. 7. The non-determinism generated by the modeling variability $\Delta\beta_s$ and the geometrical uncertainty ΔCL considered in the case-study.

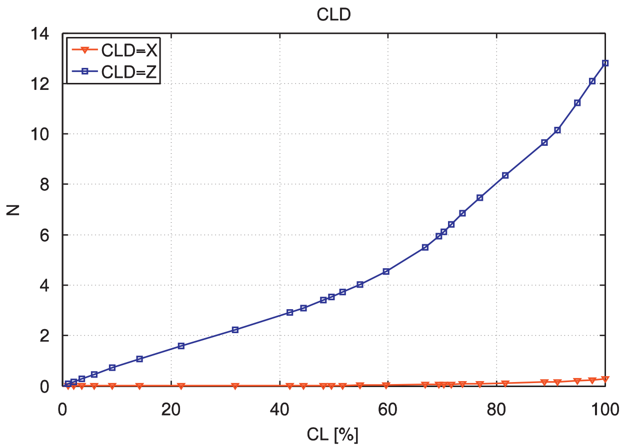

Fig. 8. Compression load deflection curve in the direction normal (CLD-Z) and tangential (CLD-X) to the contact between the plate and the weather-strip seal.

represent thus a variation of the rubber joint quasi-static tangent stiffness which follows the real CLD behavior of the joint. On the other hand, the samples along the y-axis in Fig. 7 represent a variation of the rubber joint dynamic tangent stiffness which follows the real joint rubber material dependency upon frequency and dynamic amplitude of vibration.

It is clear therefore that the adopted non-deterministic approach allows to evaluate the dependency of the dynamic behavior of a full-scale structure to the actual change in its rubber joint properties of interest, and thus from this standpoint the developed non-deterministic methodology can be considered as a large scale-sensitivity analysis tool.

5. A direct solution scheme applied to the case-study

In this section a direct solution procedure is defined where no multi-level scheme is considered and the non-deterministic approach is applied directly to the full-scale system model of the case-study. In such a procedure the weather-strip seal is modeled with a high level of detail. To investigate the sensitivity of the dynamic behavior of the clamped plate to a change in the rubber joint properties of interest a reference point frequency response function (FRF) is considered, which is evaluated in the [0–700] Hz frequency range at the bottom corner of the plate free edge.

The system response is evaluated in two steps. In a first step an analysis is performed to evaluate the system response to the pre-loaded state. In this analysis inertia effects are not considered and the analysis is a nonlinear quasi-static analysis. In a second step an analysis is performed to evaluate the system dynamic response. In this analysis the problem is linearized around the given pre-load and inertial effects are considered together with the visco-elastic material behavior, and the analysis is a linear visco-elastic dynamic analysis. To evaluate the sensitivity of the dynamic behavior of the clamped plate to the actual change in its rubber joint properties of interest, these two steps are repeated for each sample of the interval domain defined in Fig. 7.

It is assumed that the center of the two intervals represents the design nominal working conditions for the weather-strip seal, respectively, a nominal compression level of 50 percent, and a visco-elastic material behavior characterized by a nominal strain amplitude of the excitation signal of 1 percent. The deformation of the detailed FE model of the component for the nominal compression level CL = 50 percent is shown in Fig. 9. For the nominal compression level and the nominal strain amplitude of the excitation signal of 1 percent, the nominal FRF is shown in Fig. 10. The peaks at 100, 370 and 580 Hz shown in figure, represent, respectively, the first bending, the first torsion and the second bending of the plate.

In Fig. 11 it is shown the scatter on the reference FRF due to the intervals considered on the rubber joint properties. It can be noted how the resonant peaks of the coupled system are shifted in frequency and in amplitude. The scatter generally decreases with increasing frequency. This is due to two reasons: the first reason is related to the fact that the weather-strip seal can be considered as a boundary condition for the plate, and generally the effect of boundary conditions on the global structure behavior tends to decrease as the frequency of the vibration increases; the second reason is related to the fact that for the specific problem the position of the weather-strip seal is close to the nodal line of the plate third mode.

However in such a direct solution procedure the weather-strip seal is modeled with a high level of detail, such that the plate and the weather-strip seal mass, damping and stiffness matrices are assembled in the full-scale model system matrices and the vibration problem is analyzed as a whole. As the inclusion of a detailed model of the joint into a full-scale system model is already computationally expensive, predicting the effect that an interval description of the material properties have on the full-scale system behavior becomes impractical for large problems, and alternative strategies are

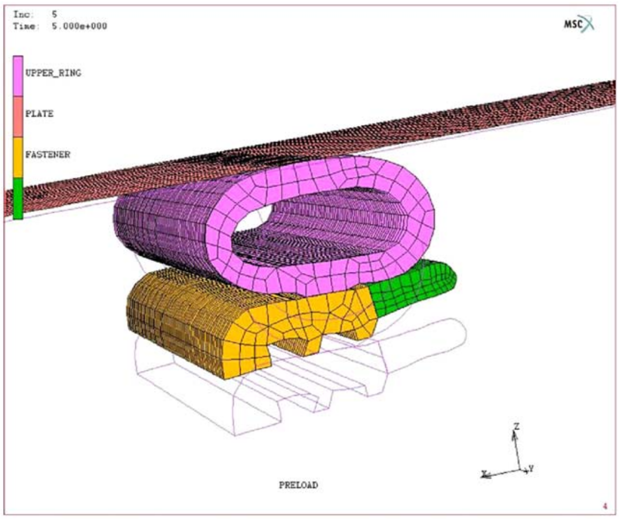

Fig. 9. System deformation for CL = 50 percent.

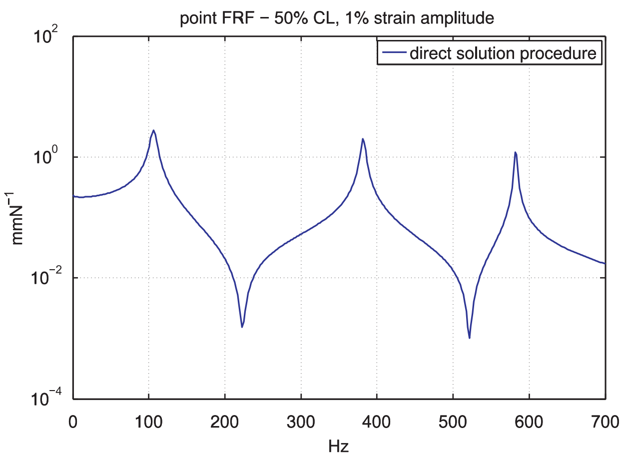

Fig. 10. The FRF calculated with the direct solution procedure for the nominal conditions.

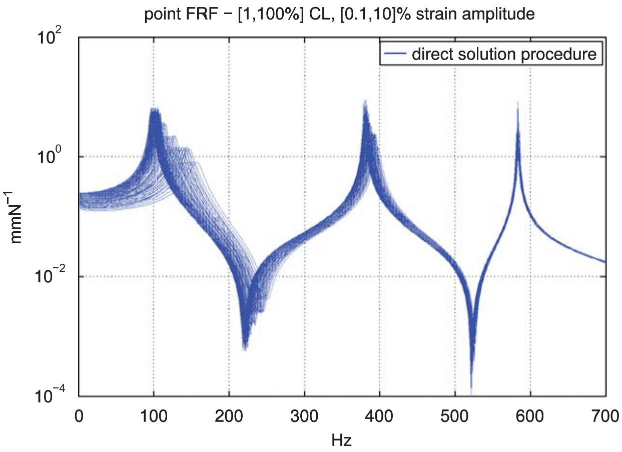

Fig. 11. The FRFs calculated with the direct solution procedure when non-determinism is considered.

needed. In Section 6 it is presented the multi-level scheme developed in this study to reduce the computational burden introduced by the non-deterministic approach.

6. The three-level solution scheme applied to the case-study

In this section the developed three-level methodology is applied to the case-study. The developed multi-level approach allows to reduce the computational burden introduced by the non-deterministic approach by reducing the contribution to the full-scale system of the mass, stiffness and damping matrices of the detailed weather-strip seal FE model. This is done by introducing an equivalent linear interval parametric rubber joint model, ready to be assembled in a full-scale system model at a reasonable computational cost. When applying the developed three-level solution procedure to the case-study, the problem is split into three different level of analysis: the material, the component and the system level.

6.1. The material level

When considering the intervals defined on the weather-strip seal pre-load and on the weather-strip seal rubber material dynamic amplitude, it should be noted that in the material level only the uncertainties and variability related to the rubber joint material needs to be considered. The identification and the definition of the material parameters which need to be described in a non-deterministic sense is done as in Section 4. In the material level only the interval on the parameter $\Delta C_i = \Delta\beta_s$ is identified, which is represented by the samples in the y-axis direction of Fig. 7.

6.2. The component level

As described in Section 2, the component level consist of two steps: in the first step a detailed model of the component is generated and a component property of interest is identified. The intervals identified at the material level ΔC_i are propagated to the component level, and according to the geometrical specifics of the problem, additional intervals ΔG are generated at component level and translated into the intervals on the component properties of interest. In the second step, an equivalent parametric linear model for the component is generated, and the intervals which have been identified on the detailed component are translated into the intervals Δk_i on the parameters describing the equivalent model.

Since the weather-strip seal runs along the plate width, it is noted that the case-study can be considered as a system with a rubber joint distributed along a line. If the interest is in the low frequency behavior of the weather-strip seal (i.e. below the weather-strip seal resonant behavior), the rubber joint can be reduced at component level to a single bi-dimensional section of nominal wall thickness, and further on the mechanical characteristic of this nominal section can be studied and then translated into the equivalent linear interval parametric model. The reduction from a three-dimensional to a bi-dimensional FE analysis at component level and further on to an equivalent linear interval parametric model of the bi-dimensional section strongly reduces the overall solution computational burden.

It is noted from the CLD-X curve in Fig. 8 that in the direction tangential to the contact region between the plate and the weather-strip seal there is no significant component stiffness change up to 90 percent, and thus the rubber joint vertical stiffness in the direction normal to the contact is sufficient to describe the component influence on the system behavior. The detailed FE model of the weather-strip seal single bi-dimensional section of nominal wall thickness is shown in Fig. 12. To extract the section mechanical characteristic a one degree of freedom system approximation is considered, where a reference mass vibrates on a spring of complex stiffness (i.e. the weather-strip seal). A point FRF is numerically simulated,

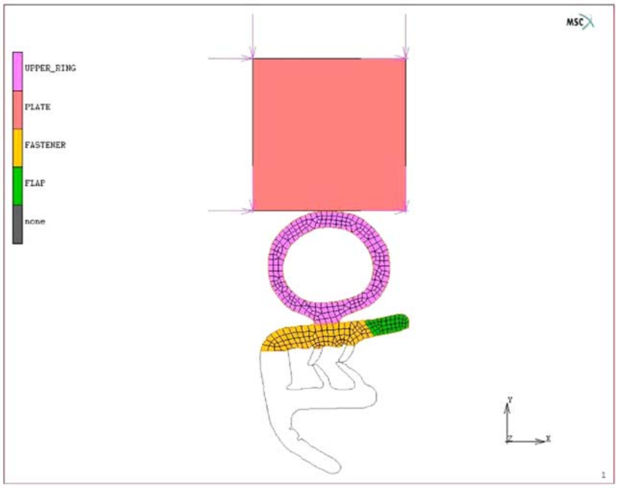

Fig. 12. The rubber joint is reduced to a bi-dimensional component section of nominal wall thickness, which is analyzed for its dynamic characteristic.

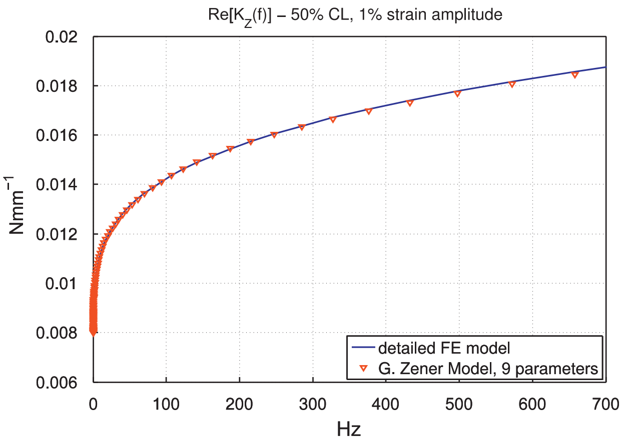

Fig. 13. Real part of the component vertical stiffness K_Z, comparison between the detailed FE component model and the equivalent generalized Zener model in nominal conditions.

from which the section complex stiffness $K^*(\omega)$ is extracted considering that

$$K^*(\omega) = K + jH \tag{8}$$

and that K and H are related to the real R and imaginary I part of the numerically simulated point FRF in the following way:

$$K = \omega^2\left(\frac{R}{R^2+I^2} + M\right) \tag{9}$$

$$H = \omega^2\left(\frac{-I}{R^2+I^2}\right) \tag{10}$$

where M is a reference mass (when correlating with measurements the convenience of this approach is shown in [12]). For a bi-dimensional section of 1 mm nominal wall thickness the component vertical stiffness of interest K_Z for the nominal pre-load, the nominal amplitude of the vibration and as a function of frequency is shown in Figs. 13 and 14.

When material variability is considered, by evaluating the component response K_Z at each material sample, the material variability now defined in terms of variability on the parameters of the chosen material model is propagated to the component level to give the scatter on the component vertical stiffness ΔK_Z. When also the geometrical modeling uncertainty $\Delta G_{pr} = \Delta \mathrm{CL}$ is considered, the full scatter domain on the component vertical stiffness ΔK_Z is evaluated. In Figs. 15 and 16, the full scatter on the real part and on the loss factor of the component vertical stiffness is shown.

It is interesting to note that with the multi-level approach the frequency resolution used to characterize the real and the loss factor part of the mechanical characteristic of the component vertical stiffness K_Z can be chosen much lower than the frequency resolution needed to evaluate the full-scale system resonant behavior. Generally a high resolution is needed to

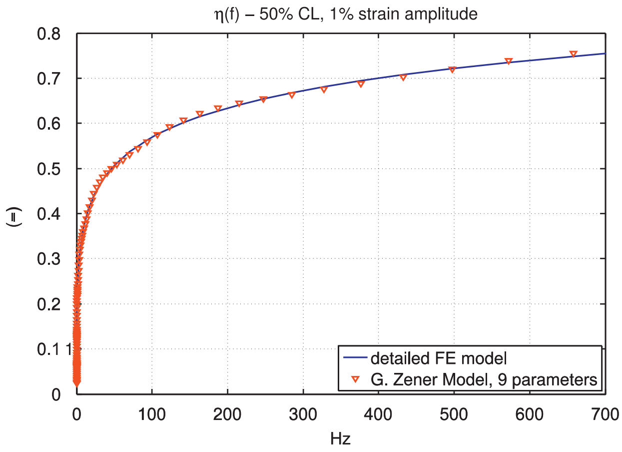

Fig. 14. Loss factor of the component vertical stiffness K_Z, comparison between the detailed FE component model and the equivalent generalized Zener model in nominal conditions.

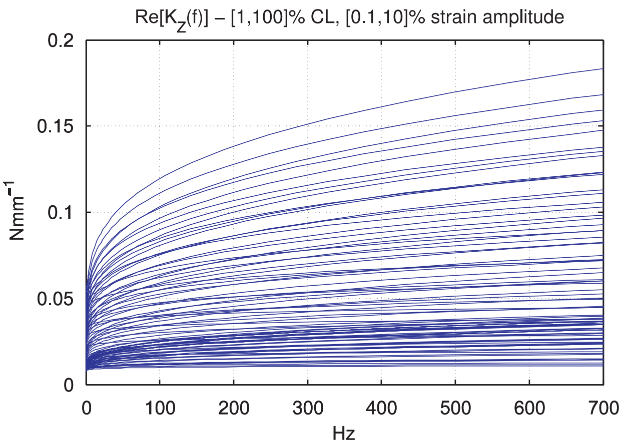

Fig. 15. Scatter on the real part of the component vertical stiffness K_Z when the modeling variability and the geometrical uncertainty are considered.

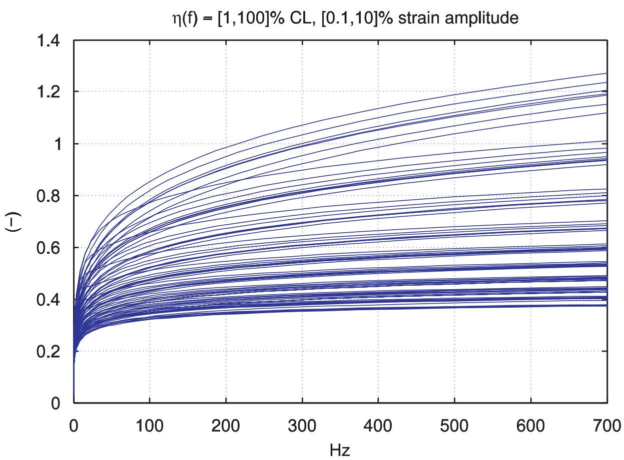

Fig. 16. Scatter on the loss factor of the component vertical stiffness K_Z when the modeling variability and the geometrical uncertainty are considered.

describe in detail the full-scale system behavior around its resonant peaks. This is not the case for the component where a much lower frequency resolution is sufficient to describe its behavior. However, the reduction of the frequency resolution is possible only if the component has no resonant behavior in the frequency range of interest, as it is assumed in this case.

Once the equivalent model is fitted and assembled at system level, the assembled equivalent full-scale system can be resolved back with the required higher frequency resolution.

In the second step of the component level an equivalent parametric model is fitted to the component vertical stiffness ΔK_Z. When modeling the dynamic behavior of elastomeric connections, a commonly applied mechanical solid model is the standard linear model [33], also know as the Zener model. The standard linear model is a mechanical solid model of visco-elastic behavior consisting of a spring which is in parallel to a series connection of a spring and a dashpot. According to the assumed material visco-elastic behavior, a generalized Zener model with nine parameters is chosen. The generalized Zener model is obtained by adding in parallel to the Zener model three series connections of a spring and a dashpot, as shown in Fig. 17. The mechanical characteristic of this model is given by Eq. (11). The meaning of the parameters K_i and $\tau_i = C_i/K_i$ is clear from Fig. 17:

$$\begin{aligned} K^*(\omega) &= K_{\text{storage}}(\omega) + \mathrm{j}K_{\text{loss}}(\omega) \\ &= K_0 + K_1\frac{\omega^2\tau_1^2}{1+\omega^2\tau_1^2} + K_2\frac{\omega^2\tau_2^2}{1+\omega^2\tau_2^2} + K_3\frac{\omega^2\tau_3^2}{1+\omega^2\tau_3^2} + K_4\frac{\omega^2\tau_4^2}{1+\omega^2\tau_4^2} \\ &\quad + \mathrm{j}\left[K_1\frac{\omega\tau_1}{1+\omega^2\tau_1^2} + K_2\frac{\omega\tau_2}{1+\omega^2\tau_2^2} + K_3\frac{\omega\tau_3}{1+\omega^2\tau_3^2} + K_4\frac{\omega\tau_4}{1+\omega^2\tau_4^2}\right] \end{aligned} \tag{11}$$

A nonlinear least-squares data-fitting problem is set up to find the coefficients of Eq. (11) defining the generalized Zener model that best fits the mechanical characteristic of the detailed FE model of the weather-strip seal single bi-dimensional section of nominal wall thickness. For the nominal conditions the real and the loss factors of the mechanical characteristic of the fitted generalized Zener model are shown in Figs. 13 and 14.

In Fig. 18 it is shown the resulting scatter on the parameters defining the fitted generalized Zener model, Δk_i and Δc_i, when non-determinism is considered. It is interesting to interpret the spreading on the equivalent model parameters. The parameter K_0 is directly related to the quasi-static behavior of the component. A variation of K_0 describes a variation of the component geometrical tangent stiffness correspondent to a variation of the component pre-load: the value of K_0 is

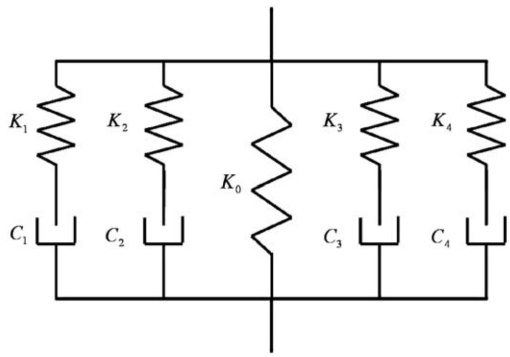

Fig. 17. The generalized Zener model used to fit the component vertical stiffness K_Z.

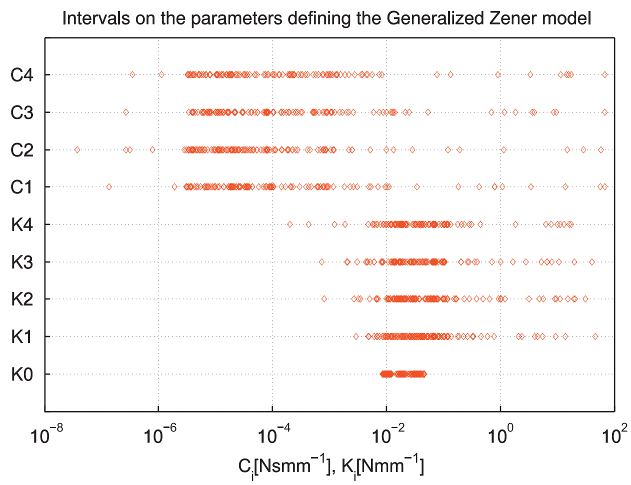

Fig. 18. Scatter on the parameters defining the generalized Zener model used to fit the component vertical stiffness K_Z.

determined by the tangent to the CLD-Z curve in Fig. 8 at the corresponding CL. The interpretation of the spread on the other equivalent model parameters is less apparent as it results from the fitting procedure. Whether the K_s and the C_s seem to have a Gaussian distribution with some skewness, it needs to be considered that the values of the K_s and the C_s are not unique (i.e. the same K_Z curve can be represented by a Zener model with different set of parameters) and thus that the chosen fitting procedure is influencing their distribution.

6.3. The system level

At system level the weather-strip seal is substituted with the previously developed equivalent linear interval parametric model opportunely distributed, as shown in Fig. 19. It is recalled that the case-study can be considered as a system with a rubber joint distributed along a line, since the weather-strip seal runs along the plate width. The behavior of the whole weather-strip seal can be considered equivalent to the behavior of a set of equivalent complex springs acting in parallel and distributed on the plate along the line of action defined by the weather-strip seal contact region. The generalized Zener model is thus distributed along the plate width according to the formula:

$$K_e^* = K_{1\,\mathrm{mm}}^* * L/N \tag{12}$$

where K_e^* is the mechanical characteristic of a single FE generalized Zener element; $K_{1\,\mathrm{mm}}$ is the mechanical characteristic of the generalized Zener model fitted on a weather-strip seal bi-dimensional section of 1 mm nominal wall thickness; L is the plate width and N is the number of FE generalized Zener elements distributed along the plate width.

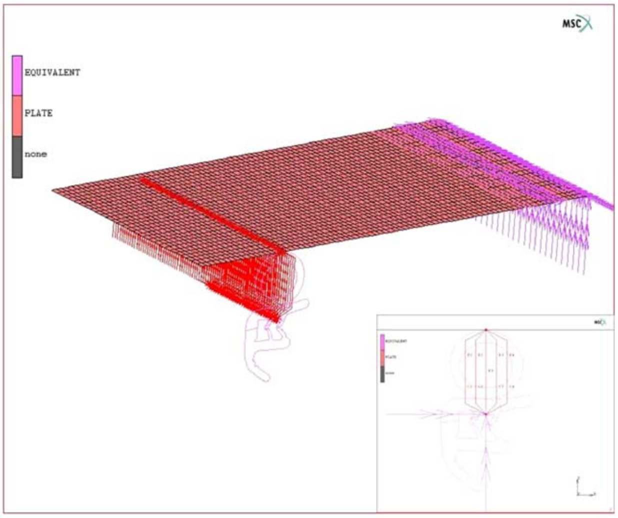

Fig. 19. The FE generalized Zener model assembled to the full-scale system.

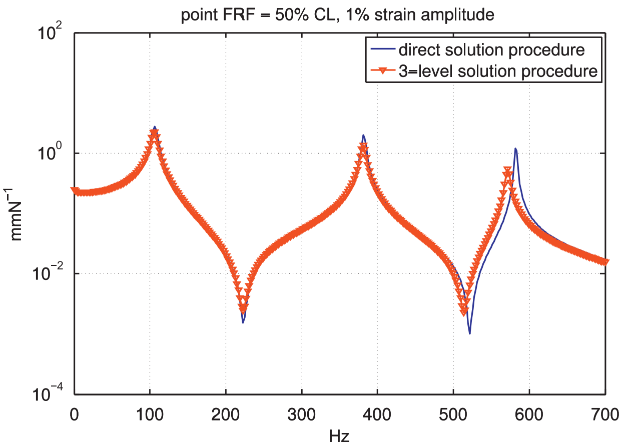

Fig. 20. Comparison between the direct solution and the developed methodology applied on the system in nominal conditions.

In Fig. 20, a comparison between the point FRF calculated with the direct solution procedure and the three-level solution procedure for nominal conditions is shown. The first two FRF resonant peaks predicted with both procedures are the same, while the plate third resonant peak predicted by the three-level methodology is slightly lower than the same resonant peak predicted with the direct solution procedure applied on the full-scale system model with the weather-strip seal modeled in detail. This difference results from the combination of two effects: the first effect is related to the fact that the plate third mode is the plate (second) bending mode which shows a nodal line close to the line of action defined by the contact region between the weather-strip seal and the plate. As a consequence for this mode, the plate rather than vibrating on the weather-strip seal tends to vibrate around it. The second effect is related to the fact that since the generalized Zener model is a point-to-point connection to the plate along this line of action, the closer this line is to the plate third mode nodal line the less the effect of the equivalent model.

When considering the non-determinism, it should be noted that the intervals identified on the parameters defining the generalized Zener model, Δk_i and Δc_i, are not independent. Sampling independently on these intervals, has two effects: the sampling domain increases, and the correlation between the parameters is lost. While for a such limited amount of intervals the first effect can be negligible, the second effect instead can be important. The correlation between the parameters is principally through the material model, and ignoring such a correlation could lead to non-physical solutions, and to an overestimation of the bounds on the system response. In Section 6.2 it is shown that at component level a generalized Zener model is fitted for each sample of the domain defined by the interval on the component property of interest ΔK_Z. To avoid non-physical solutions, rather than re-sampling on the domain defined by the intervals Δk_i and Δc_i, the fitted generalized Zener model is directly assembled at system level, and the scatter on the assembled equivalent system model is thus evaluated directly from the domain defined by the interval on the component property of interest ΔK_Z.

In Fig. 21 it is shown a comparison between the scatter on the point FRF calculated with the direct solution procedure and the three-level solution procedure (for readability the scatter is substituted with its envelope). Apart from the third peak, the scatter predicted with both procedures is substantially the same. As explained previously, the difference in the third peak is related to the relative position of the mode nodal line to the position of the line of action defined by the plate-seal contact region, and to the point-to-point nature of the equivalent model. It should be noted also that at higher compressions the contact region between the weather-strip seal and the plate cannot be approximated anymore as a contact line but needs to be considered as a contact area. In addition at higher compressions the stiffness of the component in the direction tangential to the contact might not be considered negligible anymore. Apart from including the model of the component tangential stiffness, the previous issues can be solved by distributing the equivalent elements also along the plate length direction in agreement with the contact surface for an average level of compression, or by using specific multi-point constraint elements to distribute the effect of the equivalent model to a wider region on the plate (see, e.g. the work of Lardeur et al. [34]).

The small difference observed in the amplitudes between the direct solution procedure and the three-level solution procedure can be explained instead by considering that the nonlinear least-square data-fitting problem which is set up to find the parameters of the generalized Zener model is an automatic procedure which gives for each K_Z the optimal set of parameters according to a stopping criteria based on a threshold on the residual value. Until the residual becomes lower than the threshold, a new starting point for the fitting procedure is produced. Since the residual is a scalar which sums up the errors on each frequency line in the frequency band of interest, it is likely to have regions in frequency better fitted than others. Fitting with an appropriate weighting function over the frequency band of interest could resolve this issue.

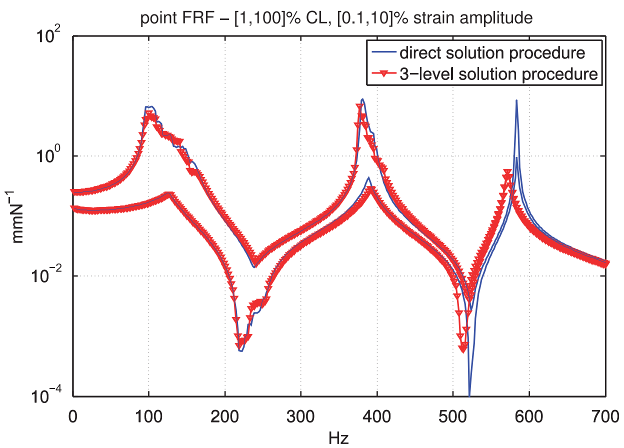

Fig. 21. Comparison between the scatter on the point FRF calculated with the direct solution procedure and the three-level procedure when non-determinism is considered (for readability the scatter is substituted with its envelope).

The total computational time required for solving the component FE detailed model, fitting the equivalent linear interval parametric model and solving the equivalent system model, was less then 20 percent of the total time needed for solving the full-scale system with the direct solution procedure.

7. Conclusions

In this study a non-deterministic methodology has been presented based on a three-level approach which aims at evaluating directly in the frequency domain the influence of the rubber joints on the NVH response of a full-scale system model, when rubber nonlinear visco-elastic behavior is considered.

It was observed that the specific strain amplitude of the vibration a rubber joint experiences in real-life conditions is generally not known by the design engineer, and this lack of knowledge often does not justify the computational burden and the modeling effort of including detailed nonlinear models of the joint in a full-scale system model.

By observing the bounded nature of the rubber material storage modulus in function of the strain amplitude of the input vibration, rather than including a representation of the rubber nonlinear visco-elastic behavior directly in the model, it is proposed to model the material nonlinear visco-elastic behavior by using a linear visco-elastic material model defined in an interval sense. It is shown that in this manner a linear visco-elastic approach can be used in each interval and the sensitivity of the NVH response of the full-scale system model to a change in its rubber joint material properties can be evaluated.

It is shown that the adopted non-deterministic approach allows to evaluate the dependency of the dynamic behavior of the full-scale structure to the actual change in its rubber joint properties of interest, and thus from this standpoint the developed non-deterministic methodology can be considered as a large-scale sensitivity analysis tool.

The advantage of this approach is twofold: it allows to work directly in the frequency domain; and it allows to use commercial FE codes where linear visco-elasticity is generally implemented.

To reduce the computational burden introduced by the non-deterministic approach, a three-level solution approach based on a material, component and system level is presented. It is shown that the developed three-level approach allows to reduce the computational burden introduced by the non-deterministic approach by reducing the contribution to the full-scale system of the mass, stiffness and damping matrices of the detailed weather-strip seal FE model. This is done by introducing an equivalent linear interval parametric rubber joint model, ready to be assembled in a full-scale system model at a reasonable computational cost.

By using a commercial finite element code the developed methodology is illustrated through a numerical case-study which represents a typical NVH automotive problem: the low-frequency dynamic analysis of automotive door weather-strip seals. The developed non-deterministic three-level methodology is compared with a direct solution procedure in terms of accuracy and computational effort. In the direct solution scheme the non-deterministic approach is applied directly to the full-scale system model of the case-study with the weather-strip seal modeled in detail. It is shown that the total computational time required for solving the component FE detailed model, fitting the equivalent linear interval parametric model and solving the equivalent system model, is less then 20 percent of the total time needed for solving the full-scale system with the direct solution procedure.

It is important to note that the developed methodology can be applied only when the rubber joints of interest work below their proper resonant frequency range. However this is the case for most of the rubber joints, which are designed to work far from their natural frequencies.

In addition the multi-level design of experiment approach used to propagate the intervals through the material, the component and the system levels does not guarantee the estimation of exact bounds in the full-scale system NVH response. However the general behavior of the system can be predicted and the correlation between the system parameters investigated.

The developed non-deterministic three-level methodology represents a consistent first step in the development of virtual prototyping and product refinement engineering tools for the modeling of the NVH behavior of rubber joints in real-life applications.

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